Structured Proofs

Simple Induction

By induction on `\var : \Ty`:

| Base case: ? |
| Induction step: ? |
| ... Induction hypothesis ... |

Making base case, induction step, and induction hypothesis explicit:

By induction on `\var : \Ty`:

| Base case '?': ? |
| Induction step '?': ? |
| ... Induction hypothesis '?' ... |

Remember that in nested inductions, induction hypotheses always need to be made explicit!

Induction pattern for sequences (choose x wisely!):

Theorem: `P`
Proof:
By induction on `\xs : \Seq \A`:

| Base case `P[\xs = \es]`: ? |
| Induction step `\forall x : \A \cdot P[\xs = x < \xs]`: For any `\x`: ? |

These can also be used for proving theorems of shape

`\forall \var : \Ty \cdot P` 
by induction on precisely that universally-quantified variable, that is, `\for \var : \Ty`:

The induction hypothesis is then `P`.

Example for sequences:

Theorem: `\forall \xs : \Seq \A \cdot P`
Proof:
By induction on `\xs : \Seq \A`:

| Base case `P[\xs = \es]`: ? |
| Induction step `\forall x : \A \cdot P[\xs = x < \xs]`: For any `\x`: ? |

Facts that can be shown by “Evaluation”

Only where enabled (and never can contain variables):

Fact `6 \cdot 7 = 42`,
Fact `6 > 7 \equiv false`.

Assuming the Antecedent

Assuming `\p`, `\q`:

| ? |
| ... Assumption `\p` ... |

Assuming `\p` and using with ...

| ... Assumption `\p` ... |

Assuming a Witness

Assuming witness \(x\) satisfying \("P":")

Proven for \(Q\) using Assumption `\p` 

```
Assuming witness `x` satisfying `P`:
Proof for `Q` using Assumption `P`
```

proves \(\exists x \cdot P\Rightarrow Q\), provided \(-\text{occurs}(x', `P`).

Assuming `x` satisfying `P` by Hint:

Proof for `Q` using Assumption `P`

```
Assuming `x` satisfying `P` by Hint:
Proof for `Q` using Assumption `P`
```

proves `\(Q\)` if the hint proves `\(\exists x \cdot P\)`, provided \(-\text{occurs}(x', `P`).

Proving Universal Quantifications

Proving `\forall v : \N \cdot P`:

| For any `v : \N`:
| \(\text{Proof for } P\) |

Proving `\forall v : \N \mid R \cdot P`:

| For any `v : \N` satisfying `R`:
| \(\text{Proof for } P\) using Assumption `R` |

Case Analysis

By cases: `\p`, `\q`, `\r`
Completeness:

| ? |
| Case `\p`:
| ? |
| ... Assumption `\p` ... |

Continuing a calculation with a structured proof for the last calculation expression:

```
Continuing a calculation with a structured proof for the last calculation expression:
```

is the same, but with different indentation!, as:

```
Continuing a calculation with a structured proof for the last calculation expression:
```

Subproofs

```
Subproof for `\ldots`:
\text{Proof for this:}
\text{Proof for } `P`\)
```

Theorems Used as Proof Methods (Example)

```
Using “Mutual implication”: Subproof for `\ldots \Rightarrow \ldots`?
Subproof for `\ldots \Rightarrow \ldots`?
```

Side Proofs

Side proof for `\p`:

| ? |
| Continuing with goal `\?`:
| ? |

(Multiple side proofs at the same indentation are possible, and can use any previously-established local property.)

Disabling Hints Producing Time-outs

Add “;" at the beginning of the hint:

```
\equiv( ?, “Golden rule” )
```

2019-12-20 — CalcCheck Syntax Hints
Selected CalcCheckWeb Key Bindings
(See Getting Started with CalcCheckWeb for the complete listing.)

The following key bindings work the same in both edit and command modes:

Ctrl-Enter performs a syntax check on the contents of all code cells before and up to the current cell.

Ctrl-Alt-Enter performs proof checks (if enabled) on the contents of all code cells before and up to the current cell. During Midterm 1: Same as Ctrl-Enter.

Shift-Alt-RightArrow enlarges the width of the current code cell entry area by a small amount.

Ctrl-Shift-Alt-RightArrow enlarges the width of the current code cell entry area by a large amount.

Shift-Alt-LeftArrow reduces the width of the current code cell entry area by a small amount.

Ctrl-Shift-Alt-LeftArrow reduces the width of the current code cell entry area by a large amount.

Ctrl-Shift-v (for visible spaces) toggles display of initial spaces on each line as "_" characters.

ONLY if you are logged in via Avenue:

Ctrl-Shift-s saves the notebook on the server.

To be safest, use in command mode, e.g. after clicking on the area of a code box where the line number would be displayed.

Check the pop-up whether it is the CalcCheckWeb pop-up saying "...Notebook saved to ...". (Links for reloading the last three saved versions are displayed when you view the notebook again.)

In edit mode, you have the following key bindings:

Esc enters command mode

Alt-i or Alt-SPACE inserts one space in the current line and in all non-empty lines below it, until a line is encountered that is not indented more than to the cursor position.

Alt-BACKSPACE deletes only a space character to the left of the current cursor position, and also from lines below it, until a line is encountered that is not indented at least to the cursor position.

Alt-DELETE deletes only a space character to the right of the current cursor position, and also from lines below it, until a line is encountered that is not indented more than to the cursor position.

The last three bindings also work with Shift.

Table of Precedences

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All non-associative binary infix operators associate to the left, except **, <, =⇒, →, which associate to the right.